OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y531/01: Pure Core

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance	
1	(a)		$8-2\lambda=-6-3\mu$ **and** $-11+5\lambda=11+\mu$	**B1**	1.1a	Forming 2 correct equations in λ and μ. Could be $-2+3\lambda=8-\mu$	Any two correct equations
			$8-2\lambda=-6-3\mu$ $-33+15\lambda=33+3\mu$ $\Rightarrow -25+13\lambda=27$	**M1**	1.1	Attempt to solve (eg scaling one equation and adding or rewriting to a standard form for solution BC). Must reach an equation (possibly incorrect) with only one unknown.	$-2\lambda+3\mu=-14$ $5\lambda-\mu=22$ $3\lambda+\mu=10$
			$\lambda=4$, $\mu=-2$	**A1**	1.1	Both	
			$-2+3\times4=10$ and $8--2=10$ so they do intersect	**A1**	2.4	Checking for consistency in 3rd equation and conclusion. Equation must be correct and both sides must be evaluated Allow eg $8-2\times4=-6-3\times-2$ $0=0$ Might see $\lambda=4$ substituted into last equation and then μ being found with this.	ie $-2+3\times4=8--2$ alone is not sufficient for A1, need to see both sides becoming 10 x: $8-2\times4=0$ & $-6-3\times-2=0$ y: $-11+5\times4=9$ & $11+-2=9$
				[4]			
	(b)		(0, 9, 10)	**B1** **[1]**	1.1	Allow as vector	

Question			Answer	Marks	AO	Guidance	
2			$u = x + 1$	B1	3.1a		
			$(u-1)^3 = u^3 - 3u^2 + 3u - 1$ used in solution	M1	1.1	Attempt to expand using binomial. 4 terms.	Follow through on their $u = x + 1$
			$2x^3 + 3x^2 - 2x + 5 = 0 \Rightarrow 2(u^3 - 3u^2 + 3u - 1) + 3(u^2 - 2u + 1) - 2(u - 1) + 5 = 0$	M1	1.1	Substituting into equation. Allow if no "= 0" here. Must have an attempt at expanding $(u-1)^3$ and $(u-1)^2$	Follow through on their $u = x + 1$
			$2u^3 - 3u^2 - 2u + 8 = 0$	A1	2.5	Must be an equation	For correct equation found using sums and products of roots allow SC2 (Method required was dictated in question) Only allocate marks using main scheme, or SC method
				[4]			

Question			Answer	Marks	AO	Guidance	
3			3 + 5i is a root	B1	1.2	Need to see statement that 3+5i is a root.	May happen at end of question
			Attempt to expand $(x - (3 + 5\text{i}))(x - (3 - 5\text{i}))$	M1	1.1	Attempt to use the conjugate pair to derive a real quadratic	May see $(3 + 5\text{i})(3 - 5\text{i}) = 9 + 25 = 34$ and $(3 + 5\text{i}) + (3 - 5\text{i}) = 6$ instead of expansion
			$= x^2 - 6x + 34$ so this must be a factor	A1	2.2a		
			$x^4 - 7x^3 - 2x^2 + 218x - 1428 = (x^2 - 6x + 34)(x^2 + \ldots x - 42)$ or $(x^2 - 6x + 34)(x^2 - x + \ldots)$	M1	1.1	Attempt to factorise or divide resulting in x^2 and one other term	**NB: This question required detailed reasoning**
			$(x^2 - 6x + 34)(x^2 - x - 42)$	A1	1.1		
			$(x^2 - x - 42) = (x - 7)(x + 6) \Rightarrow$ roots -6, 7 (and 3 + 5i)	A1	1.1	3 + 5i may be mentioned as a root earlier in the solution	
				[6]			

Question			Answer	Marks	AO	Guidance	
4	(a)	(i)	Line drawn, perpendicular to line segment joining (0, -1) and (2,0) Region below line indicated as being the required region.	M1 A1 [2]	1.1 1.1	Line needs to have negative gradient with |gradient| >1 and to intersect the y axis at a positive value Exact perpendicularity not needed, but should be approximately perpendicular.	If "shading out" is used then there needs to be an indication that the required region is below the line, such as "R" placed below line or "This region" written in etc.
	(a)	(ii)	$m = -1/(½) = -2$ $4x + 2y - 3 = 0$	M1 A1 [2]	1.1 1.1	Explicitly stated	Note must be in required form $ax+by+c=0$
	(b)		Circle centre (–1, 0) radius 3 or circle centre (0, 2) radius 2. Both circles correct Correct region shaded or otherwise indicated	M1 A1 A1 [3]	1.1 1.1 1.1	Radius can be implied by axis labels or tick-marks. Region inside circle with radius 3 but outside circle with radius 2.	If M0A0 then SC1 for two circles with correct radii but centres (1, 0) and (0, –2)

Question			Answer	Marks	AO	Guidance	
5	(a)		$\mathbf{AB}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{5}{13} & -\frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}=\begin{pmatrix}-\frac{5}{13} & \frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}$	M1	2.1	BC. **AB** or **BA** correct.	Could see $\frac{1}{13}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}5 & -12\\ 12 & 5\end{pmatrix}=\frac{1}{13}\begin{pmatrix}-5 & 12\\ 12 & 5\end{pmatrix}$
			$\mathbf{BA}=\begin{pmatrix}\frac{5}{13} & -\frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}-\frac{5}{13} & -\frac{12}{13}\\ -\frac{12}{13} & \frac{5}{13}\end{pmatrix}\neq\mathbf{AB}$ so matrix multiplication is not commutative	A1 **[2]**	2.2a	BC. Other multiplication correct and conclusion	
	(b)		Rotation about O 67.4° anticlockwise	M1 A1 **[2]**	1.2 1.1	 or 1.18 rads	1
	(c)		$(\mathrm{T_B})^{-1}$ is a rotation about O by –67.4° anticlockwise (or 67.4° clockwise)	M1	3.1a	Correct inverse of their rotation $\mathrm{T_B}$.	Could also be rotation of 292.6° anticlockwise
			So $\mathbf{B}^{-1}=\begin{pmatrix}\cos(-67.4°) & -\sin(-67.4°)\\ \sin(-67.4°) & \cos(-67.4°)\end{pmatrix}$ $=\begin{pmatrix}\frac{5}{13} & \frac{12}{13}\\ -\frac{12}{13} & \frac{5}{13}\end{pmatrix}$	A1 **[2]**	1.1	or $\mathbf{B}^{-1}=\begin{pmatrix}0.385 & 0.923\\ -0.923 & 0.385\end{pmatrix}$ (allow 0.384 for 0.385)	NB: Question states "by considering the inverse transformation". SC1 For correct inverse by other method.
	(d)		det **B** = 1 and det **C** = –3	M1	3.1a	Could find **BC** and then find det(**BC**) = -3	
			So area of $N=\lvert 1\times -3\rvert\times 5=15$	A1 **[2]**	3.2a	Area must be 15, do not allow -15 or ±15	

Question			Answer	Marks	AO	Guidance	
6	(a)		$z=\frac{-(-10)\pm\sqrt{(-10)^2-4\times2\times25}}{2\times2}$	M1	2.1	Correct substitution into formula. If formula quoted allow one slip.	Or completing the square – one slip allowed.
			$z=\frac{5}{2}\pm\frac{5}{2}\mathrm{i}$	A1	1.1	Allow $z=\frac{5\pm5\mathrm{i}}{2}$ or equivalent fractions	**NB: This question required detailed reasoning**
				[2]			
	(b)		$3\omega-2=5\mathrm{i}+2\mathrm{i}\omega \Rightarrow 3\omega-2\mathrm{i}\omega=2+5\mathrm{i}$	M1	1.1	Expanding and rearranging	Must rearrange to isolate ω terms on one side and other terms on other side
			$(3-2\mathrm{i})\omega=2+5\mathrm{i}\Rightarrow\omega=\frac{2+5\mathrm{i}}{3-2\mathrm{i}}$	M1	1.1	Factorising and dividing by two term complex number	**NB: This question required detailed reasoning**
			$\omega=\frac{2+5\mathrm{i}}{3-2\mathrm{i}}\times\frac{3+2\mathrm{i}}{3+2\mathrm{i}}=\frac{6+4\mathrm{i}+15\mathrm{i}-10}{9+4}$	M1	2.1	Multiplying top and bottom by conjugate of bottom	
			$\omega=-\frac{4}{13}+\frac{19}{13}\mathrm{i}$	A1	1.1		
			Alternative method				
			$\omega=a+b\mathrm{i}\Rightarrow 3a+3b\mathrm{i}-2=5\mathrm{i}+2a\mathrm{i}-2b$	M1		Assigning real and imaginary parts, to ω expanding and rearranging	
			$3a-2=-2b$ **and** $3b=5+2a$	M1		Comparing real and imaginary parts	
			$9a-6+10+4a=0\Rightarrow a=-\frac{4}{13}$	M1		Using valid algebra to eliminate one unknown and finding the other	
			$\Rightarrow b=\frac{19}{13}\Rightarrow\omega=-\frac{4}{13}+\frac{19}{13}\mathrm{i}$	A1			
				[4]			

Question			Answer	Marks	AO	Guidance	
7			Basis Case: when $n = 1$: $2^{3n} - 3^n = 2^3 - 3 = 8 - 3 = 5$ which is divisible by 5.	B1	2.1	At least one intermediate step must be shown	
			Assume true for $n = k$ ie $2^{3k} - 3^k = 5p$ for some integer p	M1	2.1	Must have statement in terms of some other variable than n	
			$2^{3(k+1)} - 3^{k+1} = 2^3 \times 2^{3k} - 3 \times 3^k$ $= 8 \times (5p + 3^k) - 3 \times 3^k$	M1	1.1	Uses laws of indices and then inductive hypothesis properly to eliminate either 2^{3k} or 3^k (not both)	or $8 \times 2^{3k} - 3 \times (2^{3k} - 5p)$
			$= 5 \times 8p + 5 \times 3^k$ $= 5(8p + 3^k) = 5q$ for some integer q and so this is also a multiple of 5	A1	2.2a	AG. Further simplification to establish truth for $k + 1$	$5(3p + 2^{3k})$
			So true for $n = k \Rightarrow$ true for $n = k + 1$. But true for $n = 1$. So true for all integers $n \geq 1$	A1 **[5]**	2.4	Clear conclusion for induction process, following a **correct** proof by induction.	A formal proof by induction is required for full marks.

Question			Answer	Marks	AO	Guidance	
8	(a)		$(t-1)(6-t(2-2t))$ $-(t-1)((1-t)-t(2-2t))$ $+(t-1)((1-t)(2-2t)-6(2-2t))$	M1	1.1	Correct process for expanding determinant.	Fully expanded form: $2t^3+7t^2-14t+5$
			$(t-1)[(6-t(2-2t))-((1-t)-t(2-2t))+((1-t)(2-2t)-6(2-2t))]$	M1	1.1	Bringing $(t-1)$ or $(t+5)$ or $(2t-1)$ oe out as factor of the entire expression	Factors may appear BC from no working
			$(t-1)(6-2t+2t^2-1+t+2t-2t^2+2-4t+2t^2-12+12t)$ $=(t-1)(2t^2+9t-5)$ $=(t-1)(2t-1)(t+5)$	A1 **[3]**	1.1		
	(b)		$-5, \frac{1}{2}, 1$	B1 **[1]**	1.1	FT their complete factorisation of determinant into 3 linear factors.	
	(c)		$t=b^2+2$	M1	2.1	So that the system is $\mathbf{Ar}=\mathbf{c}$	
			and so $t\geq 2$ so cannot be -5, ½ or 1 therefore $\mathbf{A}^{-1}$ will exist (for all values of b) and so there will be a unique solution to the system for all values of b.	A1 **[2]**	2.4	Complete reasoning must be seen for A1.	Could test $t=1$, ½, -5 in $b^2=t-2$, and show that these do not give real values of b

Question			Answer	Marks	AO	Guidance	
9	(a)		$\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 3 \\ -16 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ -21 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}$	M1	2.1	Attempt to find the direction vector of the tunnel. Any non-zero multiple.	
			$\begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} . \begin{pmatrix} 1 \\ s \\ t \end{pmatrix} = 0$	M1	1.1	Use of $\overrightarrow{PQ}.\mathbf{b} = 0$ in the solution.	
			$-4 - 2s + 5t = 0$ $\Rightarrow 2s = 5t - 4$ $\Rightarrow s = 2.5t - 2$	A1	2.1	AG. Some intermediate work must be seen.	
				[3]			
	(b)		$\text{M} = \frac{1}{2}\left(\begin{pmatrix} -1 \\ 3 \\ -16 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ -21 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix}$	B1	1.1	Position vector (or co-ordinates) of mid-point found	
			$\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ s \\ t \end{pmatrix}$ when $z = 0$ $\Rightarrow -18.5 + \lambda t = 0$	M1	3.4	Using $z = 0$ and the equation of the line to find a 'horizontal' relationship between λ and t.	Condone errors in, or omission of, x and y components.
			$\Rightarrow \lambda = \frac{18.5}{t}$ (so $c = 18.5$)	A1	1.1		NB: Question can be answered just by considering the z coordinate. If done correctly and M1 A1 gained also allow B1 as implied.
				[3]			

Question		Answer	Marks	AO	Guidance	
	(c)	So we need to minimise $\left\lvert \frac{18.5}{t}\begin{pmatrix}1\\2.5t-2\\t\end{pmatrix}\right\rvert$	M1	3.3	Stating or implying that the length of the shaft is given by $\lvert\lambda\mathbf{b}\rvert$ and using their λ / t relationship to reduce length of shaft to a form with only one variable.	Or eg $\left\lvert \frac{18.5}{0.4s+0.8}\begin{pmatrix}1\\s\\0.4s+0.8\end{pmatrix}\right\rvert$
		$(y=)\frac{1369}{4t^2}\left(1+(2.5t-2)^2+t^2\right)$ $=\frac{1369}{4}\left(7.25-10t^{-1}+5t^{-2}\right)$	M1*	1.1	Finding expression for (squared) length of their vector	May see $\frac{37}{2}\left(7.25-10t^{-1}+5t^{-2}\right)^{\frac{1}{2}}$ Or $\frac{39701}{16}-\frac{6845}{2}t^{-1}+\frac{6845}{4}t^{-2}$ oe
		So to minimise set $\frac{dy}{dt}=\frac{1369}{4}\left(10t^{-2}-10t^{-3}\right)=0$	dep M1*	3.1a	Correct method for minimisation of (squared) length of their vector (eg differentiating and setting to 0)	Or attempt to complete the square in t^{-1}. $y=\frac{1369}{4}\left(5(t^{-1}-1)^2+2.25\right)$
		$10t^{-2}-10t^{-3}=0\Rightarrow t=1$	A1	2.2a		So min when $t^{-1}-1=0$, $t=1$
		So length of shaft $=\left\lvert 18.5\begin{pmatrix}1\\0.5\\1\end{pmatrix}\right\rvert$ or $\sqrt{\frac{1369}{4}\left(7.25-10\times1^{-1}+5\times1^{-2}\right)}$ oe	M1	3.4	Substituting their t into their form for length of shaft	
		$=18.5\times1.5=27.75$	A1	1.1		

Question			Answer	Marks	AO	Guidance	
			Alternate method:				
			$\mathbf{a} = \frac{1}{2}\left(\begin{pmatrix}-1\\3\\-16\end{pmatrix}+\begin{pmatrix}3\\5\\-21\end{pmatrix}\right)=\begin{pmatrix}1\\4\\-18.5\end{pmatrix}$	B1			
			$\mathbf{n} = \begin{pmatrix}-4\\-2\\5\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}\left(=\begin{pmatrix}-2\\4\\0\end{pmatrix}=2\begin{pmatrix}-1\\2\\0\end{pmatrix}\right)$	M1		Attempt to find normal to vertical plane containing tunnel	
			$(k)\mathbf{b} = \begin{pmatrix}-2\\4\\0\end{pmatrix}\times\begin{pmatrix}-4\\-2\\5\end{pmatrix}\left(=\begin{pmatrix}20\\10\\20\end{pmatrix}=20\begin{pmatrix}1\\\frac{1}{2}\\1\end{pmatrix}\right)$	M1		Attempt to find (multiple of) **b** by crossing their **n** with direction vector of tunnel.	
			Need $-18.5+\lambda=0 \Rightarrow \lambda=18.5$	M1		Using $z=0$ to find λ	May see multiple of **b** used eg $-18.5+2\lambda=0$
			So length of shaft $=\left\lvert 18.5\begin{pmatrix}1\\\frac{1}{2}\\1\end{pmatrix}\right\rvert$	M1			May see eg $\mathbf{r}=\begin{pmatrix}1\\4\\-18.5\end{pmatrix}+9.25\begin{pmatrix}2\\1\\2\end{pmatrix}=\begin{pmatrix}19.5\\13.25\\0\end{pmatrix}$ and then $\begin{pmatrix}19.5\\13.25\\0\end{pmatrix}-\begin{pmatrix}1\\4\\-18.5\end{pmatrix}$
			$=18.5\times 3/2=27.75$	A1			
				[6]			
	(d)		So **b** is not parallel to the z-axis so the ventilation shaft does not go straight down.	B1 [1]	3.2a	Shaft not vertical	

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